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CONTRACTION MAPPING AND ITS APPLICATION IN MENGER SPACE

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ABSTRACT
The probabilistic analogue of the Banach contraction principle as given by Sehgal and Bharucha Reid states that a contraction mapping on a complete Menger Space (*X, F, min*) has a unique fixed point ((*X*, , *min*) = (*X, F, t*) where *t*(*a*,*b*)= min{*a*,*b*}). Later it was realized that t-norm 'min' could be replaced by weaker t-norms. Sherwood showed that the above result is an exception rather than a rule: specifically for any Archimedean t-norm, there exists a complete Menger space and a contraction by Sehgal on (*X, F)* which has no fixed point. In this paper some fixed point theorem established in Menger space by using new concept of dual contraction.

INTRODUCTION
A number of authors have defined various contractive type self-mapping of metric spaces which are generalizations of well-known Banach contraction principle and have used the same technique. The contractive condition on maps produce suitable iterations, which give Cauchy sequence and a hypothesis of completeness in the range containing these sequences. These sequences produce a limit point, which becomes a fixed point of the mapping. The contractive condition on mapping has two roles; first they assure that certain iterations are Cauchy, and second, they assure the uniqueness of fixed point.

Some common fixed point theorems using sequence which are not necessarily obtained as a sequence of iterates of certain mappings are motivated by a result of Jungck [1]. He proved that a continuous self-mapping *f* of a complete metric space *(X,d)* has a fixed point provided there exists

$q \in (0,1)$ and a mapping $g : X \to X$ which commute with *f* and satisfies

(a) $g(X) \subseteq f(X)$

(b) $d(gx, gy) \le q d(fx, fy)$, for all $x, y \in X$. Then *g* and *f* have unique common fixed point.

There have been a large number of generalization of metric space.
One such generalization is Menger probabilistic space introduced in 1942 [2] by K.
Menger. A probabilistic metric space (PM space) is an ordered pair (X,F), X is a nonempty set and is mapping s.t.

(I) $F_{p,q}(x) = 1 \;\; \forall\, x > 0 \text{ iff } p = q$

(II) $F_{p,q}(0) = 0$

(III) $F_{p,q} = F_{q,p}$

(IV) $F_{p,q}(x) = 1$, $F_{q,r}(y) = 1 \Rightarrow F_{p,r}(x+y) = 1$

Where $t:[01]\times[01]\to[01]$ is a t-norm function such that t is non decreasing,

Commutative, associative and $t(a,1)=a \quad \forall \ a\in[01]$

A Menger PM space is a triple (X,F;t) where (X,F) is a PM space and t is t-norm such that $F_{p,r}(x+y)\geq t\left(F_{p,q}(x),F_{q,r}(y)\right) \quad \forall \ x,y\geq 0$

In 1960. B. Schweizer and A. Sklar have been studied these spaces in depth. These spaces have also been considered by several other authors. The first result for a contractive self-mapping on a Menger PM space was obtained by Sehgal and Bharucha Reid [3]. Let (X,F) be PM space and $f:X\to X$ be a mapping. Then f is said to contraction if $\exists \ k\in(0\ 1)$ s.t. $\forall \ p,q\in X$, $F_{f(p)f(q)}(kx)\geq F_{pq}(x)$, $x>0$. Recently Piyush Kumar Tripathi [4], [5] defined dual contraction and using to it he proved some fixed point theorems.

2.1 DEFINITION: Let (X,F,t) be a Menger space. A mapping $f:X\to X$ is called dual contraction if $\exists \ k>1$ such that $F_{fpfq}(kx)\leq F_{pq}(x)$, $x>0$

2.3 THEOREM: Let (X,F,t) be complete Menger space. Suppose $f:X\to X$ is onto and continuous mapping satisfying the condition of dual contraction. Then f has a unique fixed point.

Piyush Kumar Tripathi [4] also proved the following lemma which is used in our results.

2.1 Lemma: Let (X,F,t) be a Menger space, where t is continuous. If $\exists \ k>1$ such that $F_{fpf^2p}(kx)\leq F_{pfp}(x)$, $x>0$. Suppose $f:X\to X$ is onto mapping then $\exists$ a Cauchy sequence in X.

MAIN RESULTS

In this section, we have also prove some fixed point theorems under different contractive conditions using contraction constant $k>1$ or $k<1$.

3.1 THEOREM: Let $(X,F;t)$ be a complete Menger probabilistic metric space where $F_{p,q}$ is strictly increasing distribution function and $f:X\to X$ is continuous mapping. If $\exists\, k\in(0,1)$ s. t. $F_{f(p),f(q)}(kx)\geq \min\{F_{p,q}(x),F_{p,f(p)}(x),F_{q,f(q)}(x),F_{q,f(p)}(x),F_{f(q),f^2(p)}\}$.

Then $\exists$ a unique fixed point.

PROOF: Let $p_0\in X$. Construct a sequence $p_n = f(p_{n-1}),\ n = 1,2,3$ ………………Then

$$F_{p_n,p_{n+1}}(kx)=F_{f(p_{n-1}),f(p_n)}(kx)$$
$$\geq \min\left\{F_{p_{n-1},p_n}(x),F_{p_{n-1},p_n}(x),F_{p_n,p_{n+1}}(x),F_{p_n,p_n}(x),F_{p_{n+1},p_{n+1}}(x)\right\}$$
$$\text{i.e. } F_{p_n,p_{n+1}}(kx)\geq \min\left\{F_{p_{n-1},p_n}(x),F_{p_n,p_{n+1}}(x)\right\}$$

$$F_{p_n,p_{n+1}}(kx)\geq F_{p_{n-1},p_n}(x)\ ,x>0$$

Therefore by lemma 2.1{P_n }is a Cauchy sequency. Since (X,F,t) is complete so $p_n \to p \in X$ .Then by theorem 2.1, p is a unique fixed point of f. For uniqueness suppose $f(p) = p$, $f(q) = q$. Then $F_{p,q}(kx) = F_{f(p),g(q)}(x) \geq \min\{F_{p,q}(x), F_{p,p}(x), F_{q,q}(x), F_{q,p}(x), F_{q,p}(x)\}$ i.e. $F_{pq}(kx) \geq F_{p,q}(x)$. Which is not possible so $p = q$. Because $F_{p,q}$ is strictly increasing function and $kx < 0$

3.2 Theorem: Let ($X,F;t$) be a complete Menger probabilistic metric space where $F_{p,q}$ is strictly increasing distribution function and $f, g : X \to X$ is continuous mapping. If $\exists\, k \in (0,1)$ such that $F_{f(p),g(q)}(kx) \geq \min\{F_{p,q}(x), F_{p,f(p)}(x), F_{q,g(q)}(x)\}$ .Then f and g have a unique common fixed point.

PROOF. Let $p_0 \in X$. Construct a sequence {p_n} defined by $f(p_{2n}) = p_{2n+1}$, $g(p_{2n+1}) = p_{2n+2}$, n = 1,2,3………………..If n = 2r + 1 then

$$F_{p_n,p_{n+1}}(kx) \geq \min\{F_{p_{n-1},p_n}(x), F_{p_n,p_{n+1}}(x)\}$$

$$F_{p_n,p_{n+1}}(kx) \geq F_{p_{n-1},p_n}(x) \text{ because } F_{p,q} \text{ is strictly increasing and } kx < x$$

Again if $n = 2r$

$$F_{p_n,p_{n+1}}(kx) = F_{p_{2r},p_{2r+1}}(kx) = F_{g(p_{2r-1}),f(p_{2r})}(kx) \geq \min\{F_{p_{2r},p_{2r+1}}(x), F_{p_{2r},p_{2r+1}}(x), F_{p_{2r-1},p_{2r}}(x)\}$$

$$F_{p_n,p_{n+1}}(kx) \geq \min\{F_{p_{2r},p_{2r-1}}(x), F_{p_{2r},p_{2r+1}}(x)\}$$

then

$$F_{p_n,p_{n+1}}(kx) \geq F_{p_n,p_{n-1}}(x) \text{ , } x > 0 \text{ therefore } \forall \text{ +ve integer } n$$

$$F_{p_n,p_{n+1}}(kx) \geq F_{p_n,p_{n-1}}(x)$$

Therefore by lemma 2.1.1,{p_n}is a Cauchy sequence. Then $p_n \to p \in X$. Since $\{p_{2n+1}\}, \{p_{2n}\}$ is subsequence of $\{p_n\}$ so $p_{2n+1} \to p, p_{2n} \to p$. Then $f(p) = p$ and $g(p) = p$ that is p is common fixed point of f and g. For uniqueness suppose p and q are two common fixed-point f and g. Then, $F_{p,q}(kx) = F_{f(p),g(q)}(kx) \geq \min\{F_{p,q}(x), F_{p,p}(x), F_{q,q}(kx)\} \Rightarrow F_{p,q}(kx) \geq F_{p,q}(x)$, which is not possible because $F_{p,q}$ is strictly increasing function and $kx < x$. Therefore f and g have unique common fixed point.

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